Topic 07 – Inequalities Solutions

Section A

Q1, (Jun 2010, Q1)

Solve the inequality $3 - x < 4(x - 1)$. **[3]**

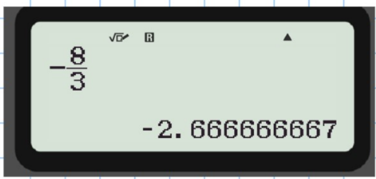
$\Rightarrow 3 - x < 4x - 4$

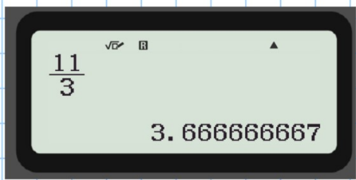
$\Rightarrow 7 < 5x \Rightarrow \frac{7}{5} < x \Rightarrow \boxed{x > \frac{7}{5}}$

Q2, (Jun 2013, Q2)

Find the integers that satisfy the inequality $-7 < 3x + 1 < 12$. **[4]**

$\Rightarrow -8 < 3x < 11 \Rightarrow -\frac{8}{3} < x < \frac{11}{3}$





$\Rightarrow \boxed{x = -2, -1, 0, 1, 2, 3}$

Q3, (Jun 2014, Q1)

Solve the following.

$$-6 < 2x - 1 < 7$$ **[3]**

$\Rightarrow -5 < 2x < 8 \Rightarrow \boxed{-\frac{5}{2} < x < 4}$

Q4, (Jun 2016, Q1)

Solve the inequality $1 - 2(x - 3) > 4x$. **[3]**

$\Rightarrow 1 - 2x + 6 > 4x \Rightarrow 7 - 2x > 4x$

$\Rightarrow 7 > 6x$

$\Rightarrow \frac{7}{6} > x \Rightarrow \boxed{x < \frac{7}{6}}$

Q5, (Jun 2017, Q1)

Solve the inequality $-2 < 3x + 1 < 7$. **[3]**

$\Rightarrow -3 < 3x < 6 \Rightarrow \boxed{-1 < x < 2}$

Q6, (Jun 2018, Q1)

Solve the inequality $2 - x < 1 + 3(x - 2)$. **[3]**

$\Rightarrow 2 - x < 1 + 3x - 6 \Rightarrow 2 - x < -5 + 3x$

$\Rightarrow 7 < 4x \Rightarrow \frac{7}{4} < x \Rightarrow \boxed{x > \frac{7}{4}}$

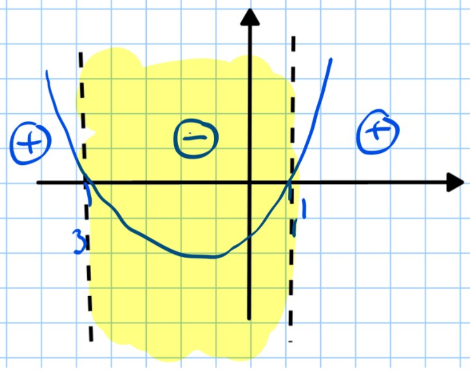
Section B

Q1 (OCR 4751, Jun 2006, Q6)

Solve the inequality $x^2 + 2x < 3$. [4]

$x^2 + 2x - 3 < 0 \quad \Rightarrow \quad (x+3)(x-1) < 0$

$\therefore$ roots are $-3, 1$

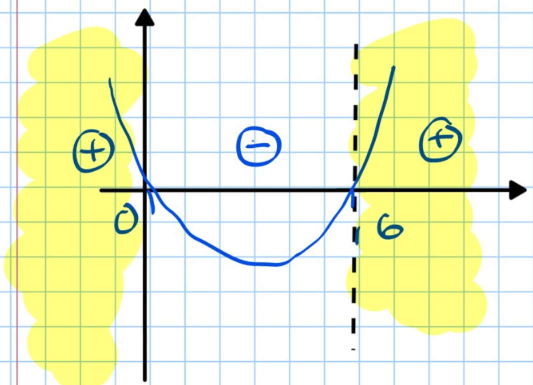


$\boxed{-3 < x < 1}$

Q2 (OCR 4751, Jun 2009, Q4)

Solve the inequality $x(x-6) > 0$. **[2]**

Roots are $0, 6$



$\boxed{x < 0 \quad \text{or} \quad x > 6}$

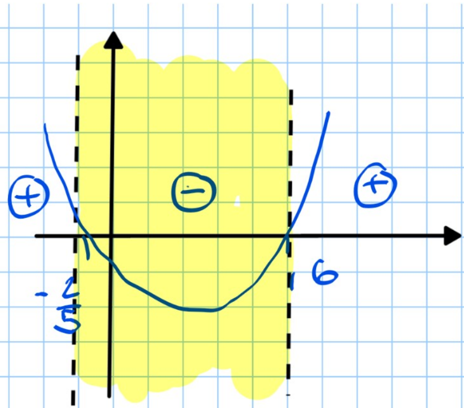
Q3 (OCR 4751, Jan 2013, Q4)

Solve the inequality $5x^2 - 28x - 12 \leqslant 0$. **[4]**

$(5x + 2)(x - 6) \leq 0$

$\therefore$ roots are $-\frac{2}{5}, \ 6$

$-\frac{2}{5} \leq x < 6$



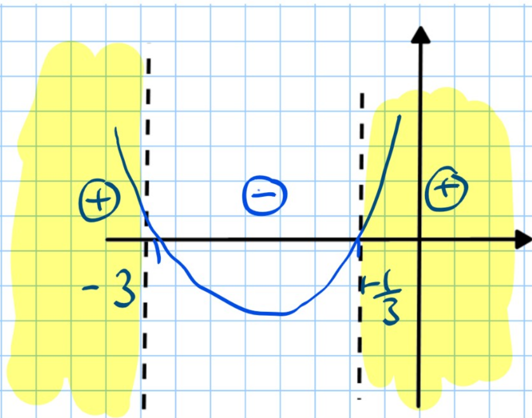
Q4 (OCR 4751, Jun 2014, Q6)

Solve the inequality $3x^2 + 10x + 3 > 0$. **[3]**

$\Rightarrow (3x + 1)(x + 3) > 0$

$\therefore$ roots are $-\frac{1}{3}, -3$

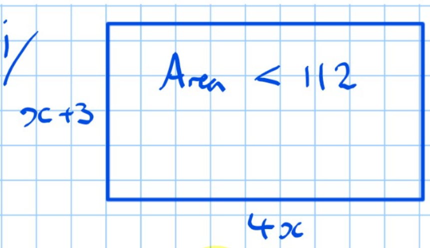
$$\boxed{x < -3 \text{ or } x > -\frac{1}{3}}$$



Q5 (OCR 4721, Jun 2012, Q9)

(i) A rectangular tile has length $4x$ cm and width $(x + 3)$ cm. The area of the rectangle is less than $112\,\text{cm}^2$. By writing down and solving an inequality, determine the set of possible values of x. **[6]**

i/



Area $= 4x(x+3) < 112$

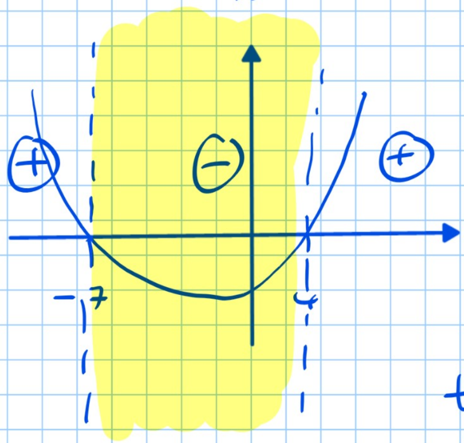
$\Rightarrow x(x+3) < 28$ $\div 4$

$\Rightarrow x^2 + 3x - 28 < 0$

$\Rightarrow (x-4)(x+7) < 0$

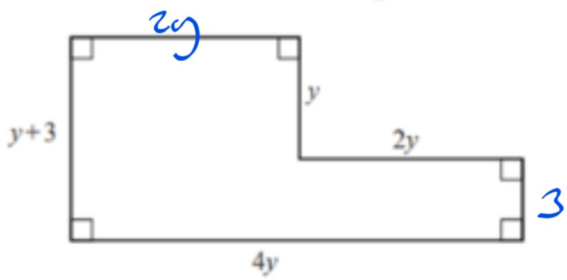
$\therefore$ roots are $4, -7$

$-7 < x < 4$ ✗



However, x cannot be ≤ 0 otherwise the side $4x$ would have negative length $\boxed{\therefore\ 0 < x < 4}$

(ii) A second rectangular tile of length $4y$ cm and width $(y + 3)$ cm has a rectangle of length $2y$ cm and width y cm removed from one corner as shown in the diagram.



Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of y. **[5]**

$P = y + 3 + 2y + y + 2y + 3 + 4y = 10y + 6$

$\Rightarrow 20 \leq 10y + 6 \leq 54 \Rightarrow 14 \leq 10y \leq 48 \Rightarrow \boxed{\frac{7}{5} \leq y \leq \frac{24}{5}}$

Q6 (OCR 4721, Jun 2005, Q8)

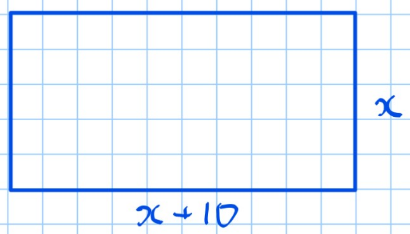
The length of a rectangular children's playground is 10 m more than its width. The width of the playground is x metres.

(i) The perimeter of the playground is greater than 64 m. Write down a linear inequality in x. [1]

(ii) The area of the playground is less than 299 m^2. Show that $(x-13)(x+23) < 0$. [2]

(iii) By solving the inequalities in parts **(i)** and **(ii)**, determine the set of possible values of x. [5]

i/



$2(x) + 2(x+10) > 64$

$\Rightarrow 2x + 2x + 20 > 64$

$\Rightarrow 4x + 20 > 64$ (Didn't ask us to solve)

ii/ Area $= x(x+10) < 299$

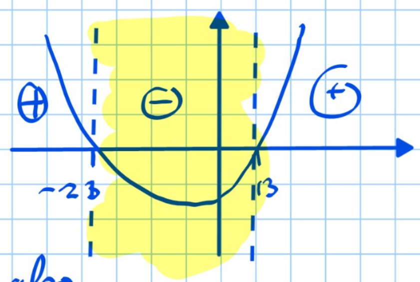
$\Rightarrow x^2 + 10x < 299 \quad \Rightarrow x^2 + 10x - 299 < 0$

$\Rightarrow (x-13)(x+23) < 0$

iii/ $4x + 20 > 64 \quad \Rightarrow 4x > 44 \quad \Rightarrow x > 11$

$(x-13)(x+23) < 0$

$\therefore -23 < x < 13$



But from linear inequality $x > 11$ also

$\boxed{\therefore \ 11 < x < 13}$